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This edition of Tech Topics takes a close look at the quality measure Cpk and related indices. Through its general acceptance, Cpk has established its value. It is important, however, to keep its limitations in mind. Jim Marshall, the author, has held numerous quality, process and development engineering positions during his career at KEMET.

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Cpk and Related Quality Indices

by J. C. Marshall

Introduction

In recent years, the managerial aspects of continuous quality improvement have received tremendous emphasis. As companies have moved toward more integrated approaches, the need for key status indicators has naturally surfaced, triggering an evolutional search for meaningful ways to quantify and monitor this critical facet of business life. Early on, familiarity with financial reporting hierarchies led to the Cost of Quality concept. Soon defect ratio reporting in Parts Per Million (PPM) gained a foothold in the electronics industry (now moving fast toward PPB, Parts Per Billion). Capability indices came about as a further refinement - an attempt to move beyond pass/fail type measures and to focus on continuous reduction in variability as a criterion for excellence.

Use of these indices has spread quickly. They have gained favor as common measures of quality, spanning departments and even industries. Major corporations embrace them both as internal monitors and as prerequisite guidelines in external communication with suppliers and customers alike. But good business decisions depend on appropriate application and interpretation of "the numbers." As with any tool, misapplication or abuse can occur. The following discussion attempts to provide some basic understanding of quality indices, and to examine a few potential pitfalls in their use, particularly in the electronics industry.

Three Important Capability Indicators: Cp, Cpk, and Cpm

Capability indices are designed to relate performance to specification in a way that provides a universally understood summation of quality. We generally say that a process is "capable" when it is "in control" and meets a given set of specifications. Whether the process is in control is determined by evaluating its average value and its variability over time. To create an index, these relevant parcels of information are mathematically packaged into one value. Among the many variants, the following three indices are probably the most recognized:

(1)
$$Cp = \frac{USL - LSL}{6\sigma}$$

where:
USL= Upper Specification Limit
LSL= Lower Specification Limit
σ = process standard deviation about the process mean

(2)
$$Cpk = \min\{CPU, CPL\}, \quad CPU = \frac{USL - \mu}{3\sigma}, \quad CPL = \frac{\mu - LSL}{3\sigma}$$

(3)
$$Cpm = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

μ = process mean
T = process target

Note that the equations for Cp, Cpk, and Cpm are scaled such that each has a value of 1.0 when μ = T = (USL+LSL)/2 and σ = (USL-LSL)/6. In a general sense, an index of 1.0 has been taken as an indication that the process is "capable." Note also that these definitions do not imply normality. This will be addressed later, but for simplicity the following illustrations reference normal distributions only.

Cp — As formula (1) shows, the value of Cp depends only on the variability of the process and the specification width. The bold curve in Figure 1 represents the 'ideal' — normally distributed output from a process with μ=T and σ =(USL-LSL)/6, resulting in a Cp value of 1.0. Because the formula does not take proximity to target into account, it cannot distinguish this condition from the less desirable ones shown as lighter curves, which also have a Cp of 1.0. In the worst-case scenario, a process operating entirely outside specification limits can have the same Cp value as one centered between the limits.

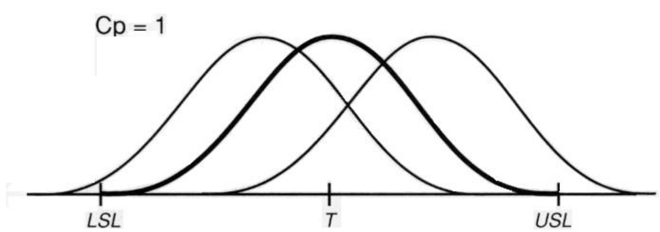

Figure 1

Despite this limitation, Cp can be effective when used as an indication of process potential. In many cases process variability presents the biggest obstacle to improvement. Once the variability is sufficiently reduced, adjusting the process mean to target is a simpler matter. In these situations Cp predicts the highest potential capability once the process is perfectly targeted.

Cpk — The Cpk index is undoubtedly the most widely acknowledged indicator in current use. It is considered superior to Cp because of its ability to judge process centering in addition to process variability. As seen in formula (2), this is accomplished by comparing the process mean to each of the specification limits. Then the lesser of these two differences is divided by the process variability to obtain the index value. This value can be negative if the mean is outside either limit.

With the promise that it accounts for both process variability and proximity to target, Cpk has been readily adapted. But this assurance can lead to some unfortunate misconceptions. In Fig. 2, we again see the "ideal" process output, μ=T and σ = (USL-LSL)/6, resulting in a Cpk of 1.0. But the other two process outputs also have Cpk values equal to 1.0. This situation can occur when improved (decreased) process variability is offset by targeting inaccuracy. Certainly all three processes are not equally desirable, but the Cpk index is unable to discriminate among them. The conscientious inclusion of the process mean in the calculation has led to another dilemma: the same estimate of Cpk can be obtained from totally different outputs having various combinations of process means and standard deviations. Not only that, but when we examine formula (2), the process target value is nowhere to be found. In reality, Cpk is a function of proximity to specification limits rather than to target. Higher Cpk values alone simply do not guarantee better targeting.

Cpm — A newer index of some interest, Cpm attempts to improve upon Cpk by including the process target value (T). As shown in formula (3), when the process mean (μ) moves further away from the target, the index diminishes in value. While this certainly places more emphasis on proximity to target, the complexity of the expression makes this index more difficult to grasp conceptually. And

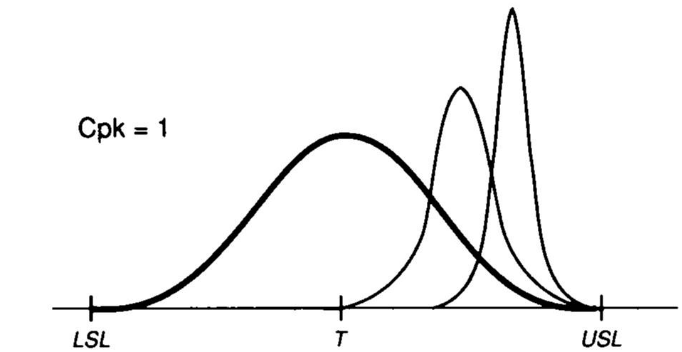

Figure 2

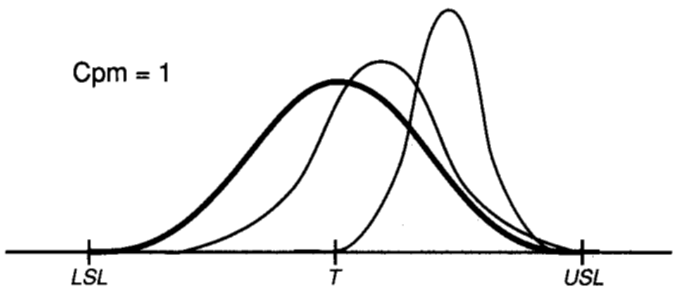

Figure 3

like the Cpk index, a given Cpm value can represent any number of process outputs, again with various combinations of mean and standard deviation values (Figure 3). Another less obvious problem quickly surfaces with processes having unilateral (one-sided) specification limits: there is no explicit target value.

To summarize to this point, three indices have been briefly described. Cp emphasizes process variability in relation to specification width. Cpk emphasizes process proximity to specification limits. Cpm emphasizes proximity to target. None of the three defines a unique process output. The remaining discussion concentrates on the limitations and applications of Cpk, currently the most popular index.

Potential Problems in Estimating the Cpk Index

Sampling Variation — Referring again to formula (2), we see that Cpk is a function of the process mean (μ), the process standard deviation (σ), and the specification limits. But in real life the true value of Cpk is hardly ever known because the true values of μ and σ are unknown. Their estimates are usually derived from sample data and then substituted into the formula to obtain a capability estimate:

(4) $$\mathbf{Cpk'} = \min\{\mathbf{CPU'}, \mathbf{CPL'}\}$$ *(apostrophe indicates an estimated value)*

where:

$$\mathbf{CPU'} = \frac{\mathbf{USL} - \overline{\mathbf{X}}}{\mathbf{3s}} \qquad \mathbf{CPL'} = \frac{\overline{\mathbf{X}} - \mathbf{LSL}}{\mathbf{3s}}$$

$$\overline{\mathbf{X}} = \frac{\sum \mathbf{Xi}}{\mathbf{n}}$$

$$\mathbf{s} = \sqrt{\frac{\sum(\mathbf{Xi} - \overline{\mathbf{X}})^2}{\mathbf{n-1}}}$$

Xi = i^{th} sample value
n = number of samples

As with all sample statistics, Cpk' varies, especially with small samples. Because Cpk' is a "folded function" based on the minimum of two values, direct mathematical prediction of its behavior is not simple. Simulation techniques, however, enable us to evaluate its sample distribution empirically. Figure 4 is a histogram of simulated results from 10,000 Cpk estimates. In this case each estimate consists of 30 samples randomly drawn from a normally distributed process having a known Cpk value of 1.0 (specifically μ=0, σ =1, USL=3, LSL=-3).

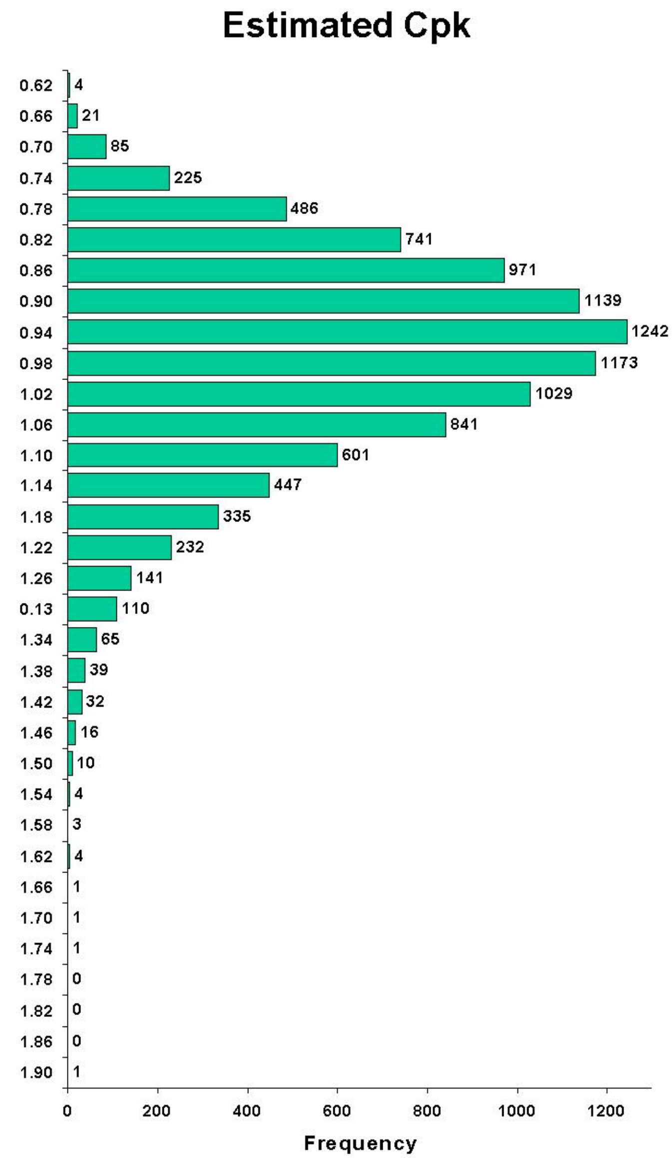

Figure 4

The resulting distribution of Cpk' values is not normal, but skewed to the high side. Neither the mean (.98) nor the median (.94) of the results is equal to the known Cpk value of 1.0, indicating that Cpk' does not provide an unbiased estimate of the true Cpk. Even more disturbing, Cpk ranges from about 38% low to 90% high, and only about half of the estimates are within 10% (high or low) of the true value. These results can be improved by increasing the sample size (or worsened by a decrease), but clearly an awareness of the effect of sampling variation on Cpk estimates is crucial to its interpretation.

Non-normal Distributions — The above results were based on sampling from a perfectly normal (bell-shaped) distribution. Further complications arise if the process output is non-normal. This is because the behavior of the sample standard deviation (and in turn Cpk') becomes more erratic. When the underlying distribution is well understood, mathematical transformations may be applied to the original data. The purpose is to "normalize" the data to obtain more consistent Cpk estimates. In the case of capacitors, several electrical parameters fall into this category for two main reasons:

a. The parameters have natural boundaries (minima, maxima) beyond which no values can occur. Examples include Direct Current Leakage (DCL) and Equivalent Series Resistance (ESR), both of which have minimum possible values determined by the laws of physics. These types of parameters invariably have unilateral (one-sided) specifications (note that these physical minima are not zero, and that to assume zero as the LSL is not a valid approach).
b. The parameters are multiplicative functions of several input variables, and are therefore rate-dependent or respond in a

proportional manner. DCL, for example, could be considered a process output that is dependent on the multiple "inputs" of field strength, temperature, dielectric thickness, counter-electrode integrity, and so on. In practice this results in a distribution which is skewed (has a long tail) away from the natural boundary.

Data Collected Over Time — Another critical concern in estimating Cpk is the time span over which the data is collected. As stated earlier, capability implies a controlled process which meets specification. The term "control" implies that performance over time is taken into account. Few processes remain in perfect control over long periods. They are subject to tooling wear, machine adjustments, changing environment, and so on. A capability estimate from data taken over a time period long enough to include these potential sources of variation can be much lower than an estimate made from short-term data.

Data from Multiple Batches — A closely related problem exists with batch processes, where sample data is summarized on a lot-to-lot basis. Pooling the summary data to obtain Cpk' will consistently overestimate the true capability, because variation over time is excluded. But combining the original raw data (not summarized) from many lots can mislead in the opposite direction. Some processes, such as those susceptible to tool wear or chemical depletion, are known to drift in only one direction over time. The real capability of these processes depends on the frequency of adjustment.

Measurement Variation — Measurement variation can also influence the Cpk estimate significantly. If the measuring system is biased (the average of repeated measurements of the same unit is different from its true value), the estimate of the process mean will also be biased. Depending on the direction of the bias, this will artificially lower or raise the Cpk estimate. Lack of precision in the measuring system (variability in repeated measurements of the same unit) always reduces the Cpk estimate.

From the preceding discussion, we begin to understand the frailty of the Cpk estimate, whose components can be seriously adulterated. Rather than being a pure estimate of the true process mean, the sample mean is also a function of measurement bias and lot-to-lot bias:

(5) $$\overline{X} = f\left\{\sigma^2_{process} + \beta_{measurement} + \beta_{lot-to-lot}\right\}$$

Likewise, the sample standard deviation is not a pure estimate of the true process standard deviation. Its squared value, called the sample variance, is a combination of variability from several sources:

(6) $$S^2 = f\left\{\sigma^2_{process} + \sigma^2_{sampling} + \sigma^2_{meas} + \sigma^2_{lots}\right\}$$

Clearly, the combined effects of sampling, measurement system, and lot-to-lot variation, as well as calculation methods, can have enormous impact on process capability estimates.

Using the Cpk Index to Estimate PPM

Great emphasis has been placed on using Cpk values as predictors of the ratio of defective units generated by a process. The electronics industry commonly expresses this ratio in Parts per Million (PPM). This emphasis developed naturally as we became more attuned to process output distributions rather than just the number of units exceeding specification. It also has roots in the happy dilemma of steadily declining defect levels – levels so low that huge amounts of real failure data are required to obtain accurate estimates. Consequently, the use of sample "variables" data (actual measurement values) to generate PPM estimates, instead of the real PPM data itself, is often the only viable approach.

Referring to the bold curve in Figure 2, note that the specification limits fall at 3 standard deviations above and below the mean. Assuming again that this is a normal distribution, we can refer to statistical tables to find the area under the curve that falls outside ±3σ. The area falling below -3σ is .135%, and the same area falls above +3σ, for a total of .27%, or 2700 PPM. We can predict that 2700 of every million units produced by this process will fall outside the specification limits.

Since 2700 PPM is rarely an acceptable defect ratio in today's competitive environment, a process with a Cpk value of 1.0 is no longer considered "capable." Other values of Cpk, such as 1.33 (63 PPM) and even 2.00(2 Parts per Billion!), have been established as new benchmarks of acceptable quality. The theoretical relationship between Cpk and PPM provides us with a handy piece of information. In practice, however, things can get out of hand quickly.

Just as a single Cpk value does not uniquely define the output distribution of a process, neither does it convey a solitary PPM ratio. As discussed earlier, the process output represented by the bold curve in Figure 2 will contain 2700 PPM defects. The other two processes, although clearly off target, actually have lower PPM levels! They have the same 1350 PPM in the tail closest to a specification limit, but virtually none of their output is expected to exceed the opposite limit. In fact, for any process output with bilateral (upper and lower) limits, PPM levels from 1350 to 2700 can be associated with a true Cpk value of 1.0. A similar relationship holds true for any value of Cpk associated with a normal distribution - the maximum possible PPM is always twice the minimum.

Add the necessity to estimate Cpk, rather than knowing its true value, and the confusion mounts. Figure 5 is a histogram of PPM estimates associated with the Cpk estimates of Figure 4. These estimates have a range of 0 - 47,608 PPM, with a mean value of 4464! The PPM estimate is within 10% of the true value less than 7% of the time. About the only merit here is that the median PPM estimate is roughly 2700.

Figure 5 again assumes a perfectly normal process output - the only gremlin is sampling variation. Non-normal process outputs only further debase the PPM estimate. Beyond the naturally occurring non-normal distributions discussed earlier, the electronics industry sometimes encounters truncated distributions. These can occur when the "tail" of the original distribution is cut off near a specification limit. The Cpk estimate will invariably be less than one, in turn predicting a high PPM level. In reality the resulting PPM level is nil.

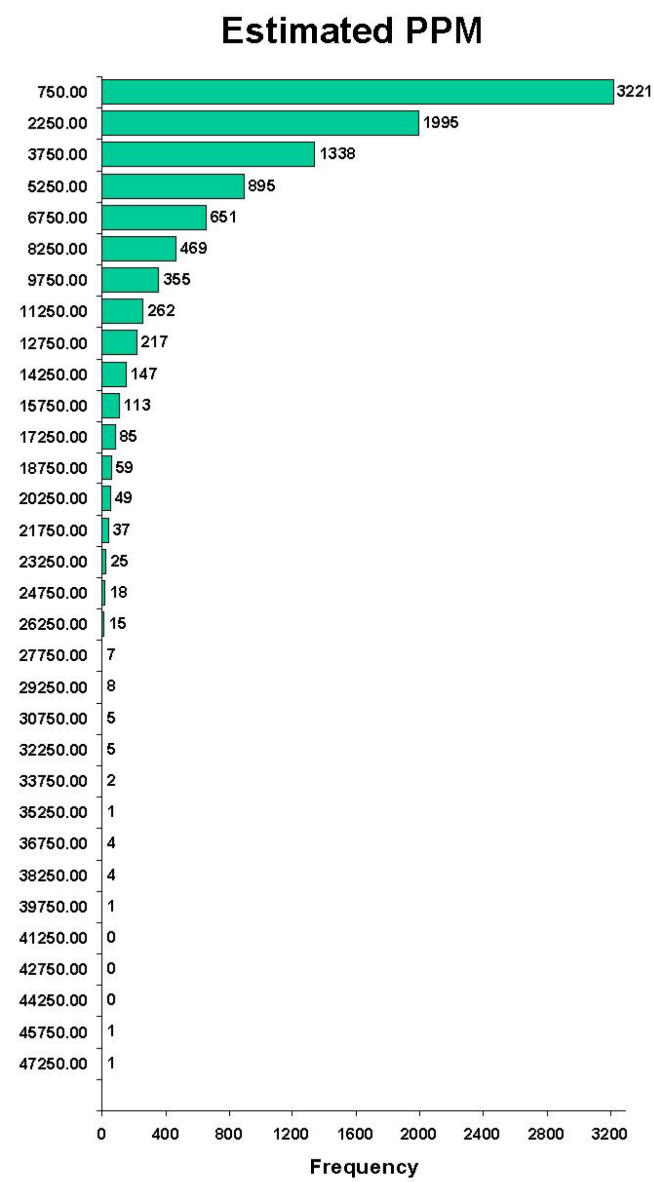

Figure 5

Additionally, certain electrical parameters are known to shift during manufacture. These effects can result when units are subjected to conditioning processes, or from natural aging phenomena. Therefore the process point where the estimate is obtained must be well defined to avoid misinterpretation. There may be rare but critical defects as well, such as open or short circuits, which have no meaningful parametric value. One way to deal with this situation is to substitute an extreme value for the critical defect, but this leads to arbitrary deflation in the Cpk estimate and obscures the underlying distribution of the remaining units. Another approach might be to employ an estimator which is robust to outliers, such as the median method (this method substitutes the median parametric value, rather than the average, for μ, and the median absolute deviation as an estimate for σ). In this case the Cpk estimate would more accurately reflect the real distribution, but the failures would be obscured. One way to avoid compromise in these situations is to use a combination of Cpk and long-term PPM assessment to properly characterize product quality.

Recommendations

The Electronic Industries Association (EIA) has published a very pragmatic guide on the use of Cpk. It specifically states that the most appropriate use of Cpk is in monitoring trends in the progress of quality improvement, not in estimating PPM. Other monitors may be just as appropriate. In the case of catastrophic failures, for instance, the actual PPM trend may be more informative. Also discouraged is the use of Cpk for comparisons among suppliers.

The guide states that the most important factor in reporting Cpk is to be consistent over time. Recommended supplemental information to be reported includes the actual values of:

1. the estimate of the mean,
2. the estimate of the standard deviation,
3. the specification limits,
4. the target value,
5. the estimate of the measurement error,
6. the shape of the underlying distribution.

"Outlier" data should be excluded only when the cause is known and permanently removed, and justification should be recorded. Recommended documentation to be maintained includes a general description of:

1. the method used to estimate the mean and standard deviation,
2. the sampling scheme (time frame, sample size, number of lots),
3. the measurement system (equipment, method, calibration).

Because the sample Cpk is neither normally distributed nor unbiased, its merit lies in its use as a reference to judge improving performance over time – not in the number itself. The EIA recommends using a non-parametric approach to evaluate trends. This can be accomplished by plotting reported values and comparing them against the median value. A conclusion could be drawn, for instance, that real improvement has taken place if four reported values in a row fall above the median value. The probability of being incorrect would be 6.25%, the same probability as getting "heads" on four straight coin tosses. This approach could also be used in monitoring other summary statistics, such as "% of lots with Cpk' above 1.33," to detect improvement over time.

Conclusion

Capability indices are often used as drivers to achieve quality goals. But because of the problems discussed, good intent can degenerate into a numbers game in which the numeric goals are either met or finessed and then forgotten, all to the detriment of continuous quality improvement. From a practical standpoint, real improvement usually begins with a basic understanding of both the targeting and the variability aspects of a situation. Which must be improved to obtain the desired outcome? The approach will be different for each; being off target will logically lead to a different approach than will the problem of too wide a spread. Reliance on Cpk estimates can easily detract from the tried and true approach that requires understanding the underlying distributional characteristics of the process output.

The real goals are to establish stability and to reduce variability wherever the potential exists for reduced cost or enhanced customer satisfaction. Capability indices are useful only when they help us to attain these goals.

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